Preface

Here are the solutions to the practice problems for my Calculus II notes. Some solutions will have more or less detail than other solutions. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Arc Length with Polar Coordinates

1. Determine the length of the following polar curve. You may assume that the curve traces out exactly once for the given range of θ.

$$r = -4\sin\theta\,,\ 0 \le \theta \le \pi$$

Step 1
The first thing we'll need here is the following derivative.

$$\frac{dr}{d\theta} = -4\cos\theta$$

Step 2
We'll need the *ds* for this problem.

$$\begin{aligned} ds &= \sqrt{\left[-4\sin\theta\right]^2 + \left[-4\cos\theta\right]^2}\,d\theta \\ &= \sqrt{16\sin^2\theta + 16\cos^2\theta}\,d\theta = 4\sqrt{\sin^2\theta + \cos^2\theta}\,d\theta = 4d\theta \end{aligned}$$

Step 3
The integral for the arc length is then,

$$L = \int ds = \int_0^\pi 4\,d\theta$$

Step 4
This is a really simple integral to compute. Here is the integral work,

$$L = \int_0^{\pi} 4\,d\theta = 4\theta\Big|_0^{\pi} = \boxed{4\pi}$$

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2. Set up, but do not evaluate, and integral that gives the length of the following polar curve. You may assume that the curve traces out exactly once for the given range of θ.

$$r = \theta\cos\theta\,,\ 0 \le \theta \le \pi$$

Step 1
The first thing we'll need here is the following derivative.

$$\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta$$

Step 2
We'll need the *ds* for this problem.

$$ds = \sqrt{\left[\theta\cos\theta\right]^2 + \left[\cos\theta - \theta\sin\theta\right]^2}\,d\theta$$

Step 3
The integral for the arc length is then,

$$\boxed{L = \int_0^{\pi}\sqrt{\left[\theta\cos\theta\right]^2 + \left[\cos\theta - \theta\sin\theta\right]^2}\,d\theta}$$

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3. Set up, but do not evaluate, and integral that gives the length of the following polar curve. You may assume that the curve traces out exactly once for the given range of θ.

$$r = \cos(2\theta) + \sin(3\theta)\,,\ 0 \le \theta \le 2\pi$$

Step 1
The first thing we'll need here is the following derivative.

$$\frac{dr}{d\theta} = -2\sin(2\theta) + 3\cos(3\theta)$$

Step 2
We'll need the *ds* for this problem.

$$ds = \sqrt{\left[\cos(2\theta) + \sin(3\theta)\right]^2 + \left[-2\sin(2\theta) + 3\cos(3\theta)\right]^2}\,d\theta$$

Step 3
The integral for the arc length is then,

$$\boxed{L = \int_0^{2\pi} \sqrt{\left[\cos\left(2\theta\right) + \sin\left(3\theta\right)\right]^2 + \left[-2\sin\left(2\theta\right) + 3\cos\left(3\theta\right)\right]^2}\, d\theta}$$